This write-up describes on how to simplify the eigenvalue decomposition of information matrix when the allocation of animals to runs is disconnected. In addition, it will also describe how the disconnectedness occurs based on the occurrence matrices.

The objective function for the second case where the first phase is randomised block design has four components to be maximised. These are the average efficiency factor of animals effect, DF associated with the residual in the between animals within runs stratum, average efficiency factor of treatment effect and DF of treatment effect. To maximise all of these components, we could combine they and compute weighted sum.

The current objective functions used when the first phase experiment is RBD is

However, these weights may not be efficient as they were determined through trial and error.

Hence, this objective function can be simplified by by performing two checks initially. This first check is to make sure the average efficiency factors associated with the animal (Within Cages) effect in the Within Runs stratum is 100%. From all the designs have found so far, the canonical efficiency factors associated with the animal (Within Cages) effect in the Within Runs stratum is always 100%. Note that the information matrix can be expressed as

where denotes eigenvalue and denotes eigenvectors. Hence, the trace of the information matrix becomes the sum of the eigenvalues which can be shown as follows

Since

where the DF is degrees of freedom associated with the Between Animals Within Cages Within Runs stratum.

Hence,

The DF associated with the Between Animals Within Cages Within Runs stratum can be computed from the projection matrices. Therefore, if the canonical efficiency factors are all ones, the sum of the canonical efficiency factors should be the same as the DF associated with the Between Animals Within Cages Within Runs stratum. This method avoids the eigenvalue decomposition and in theory should speed up the algorithm.

The second check is to make sure the treatment allocation to runs is connected, i.e. there should be v – 1 DF associated with treatment effects in the Between Animals (Within Cages) Within Runs stratum. Since we do need to compute the average efficiency factor associated with the treatment effects in the Between Animals (Within Cages) Within Runs stratum, hence the eigenvalue decomposition is required and the DF is computed from the number of the non-zero eigenvalues present.

If either of these checks failed, the objective function will give a very low value, i.e. zero, so there is still a very low chance of the design been accepted for the SA search.

The SA is implemented in two stages. The first SA stage is to determine the DF of residual. The SA is then run again with DF of residual fixed and maximise the average efficiency factor of treatment. Hence, no weights need to be determined.

**The initial design for the second case**

Confound the cage with either tag or run as the initial design.

If the total number of cages is divisible by 2, 4 and 8 (only for 8 tags), then the cages can to assigned in such way that they are completely confounded with the tags. If the number of cage is two, for the Phase 2 experiment using 4-tag system, the initial allocation of cages can be expressed as

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Runs | Tag | | | |
| 114 | 115 | 116 | 117 |
| 1 | a | a | b | b |
| 2 | a | a | b | b |
| 3 | a | a | b | b |

Otherwise the cages are always assigned to be confounded with runs. If the number of cage is three, the initial allocation of cages can be expressed as

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Runs | Tag | | | |
| 114 | 115 | 116 | 117 |
| 1 | a | b | a | b |
| 2 | b | a | b | a |
| 3 | c | c | c | c |

Note that it still need to follow the 2-by-2 structure for the same technical replicates for the initial design.

This is to minimised the number of DF associates with Animals Within Cages that may confound with either Runs and Tags by maximised the confounding between the cages with runs and tags.